Squares of Numbers

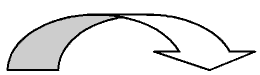
Table of Squares

Number	Square	Number	Square	Number	Square	Number	Square	Number	Square
1	1	13	169	25	625	37	1369	49	2401
2	4	14	196	26	676	38	1444	50	2500
3	9	15	225	27	729	39	1521	51	2601
4	16	16	256	28	784	40	1600	52	2704
5	25	17	289	29	841	41	1681	53	2809
6	36	18	324	30	900	42	1764	54	2916
7	49	19	361	31	961	43	1849	55	3025
8	64	20	400	32	1024	44	1936	56	3136
9	81	21	441	33	1089	45	2025	57	3249
10	100	22	484	34	1156	46	2116	58	3364
11	121	23	529	35	1225	47	2209	59	3481
12	144	24	576	36	1296	48	2304	60	3600

Although it is useful to remember all squares from 1 to 60, there is no need to memorize everything !!
The following *shortcuts* are useful:

Shortcut 1 : Squares of numbers ending in 5

For any number ending in 5, the last 2 digits of the square are always 25. For the remaining digits, look at the examples below



e.g. 35 = 3 × 4 and 25 i.e. 12*25*

65 = 6 × 7 and 25 i.e. 42*25*

115 = 11 × 12 and 25 i.e 132*25*

Shortcut 2 : Square of one number in terms of the square of the *previous* number

It is very easy to obtain $30^2 = 900$. Now if we wish to obtain 31^2, it can be obtained thus :

$31^2 = 30^2 + 30 + 31 = 961$

Similarly,

$36^2 = 35^2 + 35 + 36 = 1225 + 35 + 36 = 1296$

$66^2 = 65^2 + 65 + 66 = 4225 + 65 + 66 = 4356$

In General,

$(n + 1)^2 = n^2 + n + (n + 1)$

Q-26. *The difference between a number and two-thirds of its value is the sum of digits of the original number. How many digits could this number have ?*

☑ 1] 2 only ☐ 2] 3 only ☐ 3] 2 or 3 only ☐ 4] Any number of digits

Soln. : It obviously cannot be a single digit number (because then the number - sum of its digits = 0)
For a 2-digit number, maximum sum of digits = 9 + 9 = 18
For a 3-digit number, maximum sum of digits = 9 + 9 + 9 = 27
However, the smallest 3-digit number = 100
$^1/_3$rd of 100 (the difference by which the number exceeds its $^2/_3$rd)
= $33^1/_3$. This is more than 27. Hence it is not possible for the number to be a 3 digit number.
Similarly, for a 4 digit number, maximum sum = 9 + 9 + 9 + 9 = 36
But lowest 4 digit = 1000 $^1/_3$rd of 1000 = $333^1/_3$ which is more than 36
Not possible. Similarly for 5, 6, 7 . . . . digit numbers
Hence [1]

Q-27. *The remainder obtained when a prime number greater than 6 is divided by 6 should be ?*

☐ 1] 3 or 5 ☑ 2] 1 or 5 ☐ 3] 2 or 3 ☐ 4] 1 or 2

Soln. : The best way to solve such problems is by considering actual values and finding out which option holds true
e.g. Try 7, remainder is 1
Try 11, remainder is 5
Try 13, remainder is 1
Try 17, remainder is 5
∴ Option 2 is correct. Hence [2]
[In general, for such questions, eliminate all options which have as a remainder any factor of 6 other than 1.
∴ Option 1 has 3, options 3 and 4 have 2. Both 3 and 2 are factors of 6.
∴ They cannot be remainders when a prime number > 6 is divided by 6]

Q-28. *How many numbers can be formed using all of the digits 6, 8, 2, 1 exactly once, such that the number formed is divisible by 9 ?*

☐ 1] 16 ☐ 2] 18 ☐ 3] 24 ☑ 4] None of these

Soln. : Sum of digits = 6 + 8 + 2 + 1 = 17
∴ Any number formed using these digits will never be divisible by 9. Hence zero numbers.
Hence [4]

Q-29. *306^6 - 306 is not divisible by which of the following ?*

☐ 1] 3 ☑ 2] 4 ☐ 3] 6 ☐ 4] 9

Soln. : $306(306^5 - 1)$
306 is divisible by 3, 6 and 9
Hence [2]
Note : Such problems are best solved by elimination

∴ Since the powers are now the same value 1/12, we have to compare the base

$6^4 = 36^2 = 1296$

$8^3 = 2^9 = 512$

$3^6 = 27^2 = 729$

∴ $\sqrt[4]{8}$ is the smallest, $\sqrt[3]{6}$ is the largest.

Hence [3]

Q-53. *What does 7^{342} end in ?*

☐ 1] 1 ☐ 2] 3 ☐ 3] 7 ☑ 4] 9

Soln.: 7^1 ends in 7

$7^2 = 7 \times 7$ ∴ ends in 9

$7^3 = 7^2 \times 7$ ∴ ends in 9×7, or 3

$7^4 = 7^3 \times 7$ ∴ ends in 3×7, or 1

$7^5 = 7^4 \times 7$ ∴ ends in 1×7, or 7

$7^6 = 7^5 \times 7$ ∴ ends in 7×7, or 9

$7^7 = 7^6 \times 7$ ∴ ends in 9×7, or 3

$7^8 = 7^7 \times 7$ ∴ ends in 3×7, or 1

We find that powers of 7 follow a pattern

Power	1	2	3	(4)	5	6	7	(8)	9	10	11	(12)
Ends in	7	9	3	1	7	9	3	1	7	9	3	1
				↑				↑				↑

∴ If the power is a multiple of 4, it ends in 1
If it is 1 more than a multiple of 4, it ends in 7
If it is 2 more than a multiple of 4, it ends in 9
If it is 3 more than a multiple of 4, it ends in 3

We have 7^{342}

342/4 leaves a remainder of 2

∴ It ends in 9. Hence [4]

Q-54. *What is the value of $5^3 \times 6^3 \times 7^3$?*

☐ 1] 9426240 ☑ 2] 9261000 ☐ 3] 9120160 ☐ 4] 9864210

Soln.: In such questions, you are never expected to actually compute the answer.
What you have to look out for are certain properties of the product that will help eliminate options.
In this case, we have $5^3 \times 6^3 \times 7^3$

$5^3 = 125$

$6 = 3 \times 2$ ∴ $6^3 = (2 \times 3)^3$

∴ 6^3 will have a factor $2^3 = 8$

∴ The product will contain $125 \times 8 = 1000$

As the product has to be a multiple of 1000, it has to end in 3 zeroes at least.
Of the given options, only option [2] fulfils this requirement.
Hence [2]

Alternately : $5^3 \times 6^3 \times 7^3 = (5 \times 6 \times 7)^3$
$= (210)^3 = (21 \times 10)^3$
$= 21^3 \times 10^3 = 21^3 \times 1000$
So it has to end in 3 zeroes. Hence [2]

Q - 55. *If $m^n = 361$, 'm' and 'n' being natural numbers, which of the following could be a value of (m - n)/2 ?*

☐ 1] 9.5 ☑ 2] 180 ☐ 3] - 8.5 ☐ 4] None of these

Soln. : $m^n = 361$
Since 'm' and 'n' are natural numbers, only 2 combinations of 'm' and 'n' are possible
1) $19^2 = 361$ $\therefore m = 19, n = 2$
$\therefore (m - n) / 2 = 8.5$, none of the options
2) $361^1 = 361$ $\therefore m = 361, n = 1$
$\therefore (m - n) / 2 = 180$, option 2
Hence [2]

Q - 56. *If $5^m = 35^6 = 7^n$, what is $\dfrac{mn}{m+n}$?*

☐ 1] 5 ☑ 2] 6 ☐ 3] 7 ☐ 4] None of these

Soln. : Let $5^m = 35^6 = 7^n = k$

$\therefore 5^m = k$ $\therefore 5 = k^{1/m}$
$35^6 = k$ $\therefore 35 = k^{1/6}$
$7^n = k$ $\therefore 7 = k^{1/n}$

Now, notice that $35 = 5 \times 7$, representing this in terms of 'k' gives us

$k^{1/6} = k^{1/m} \times k^{1/n}$
$\therefore k^{1/6} = k^{1/m + 1/n}$
$k^{1/6} = k^{(m+n)/mn}$

As the base is the same, for the equality to hold, the indices should also be equal.

$$\therefore \frac{1}{6} = \frac{m+n}{mn}$$

$$\therefore \frac{mn}{m+n} = 6$$ [taking reciprocal on both sides]

Hence [2]

Q - 57. *If $p = q^{3a}$, $q = r^{4b}$, $r = p^{5c}$, find the value of abc if $p, q, r \neq 1$.*

☐ 1] 1 ☐ 2] 12 ☐ 3] 60 ☑ 4] None of these

Soln. : The first thing you observe here is that 'p' is given in terms of 'q', 'q' in terms of 'r' and 'r' in terms of 'p'.
There is a cyclic relation between 'p', 'q' and 'r'
Therefore, begin by substituting any one in terms of the other.